1. Find the order of each of the following permutations.
   (a) (1357)
   (b) (12345)
   (c) \((a_1a_2 \cdots a_k)\)

2. Write the following permutations as a product of disjoint cycles.
   (a) (1235)(413)
   (b) (13256)(23)(46512)

3. What is the order of each of the following permutations?
   (a) (124)(357)
   (b) (124)(3567)
   (c) (1235)(24567)
   (d) (345)(245)

4. Show that a function from a finite set \(S\) to itself is one-to-one if and only if it is onto. Is this true when \(S\) is infinite?

5. How many elements of order 5 are there in \(S_7\)? (Hint: consider the cycle structure of such an element.)

6. Let \(G\) be a group of permutations on a set \(X\). Let \(a \in X\) and \(\text{stab}(a) = \{\alpha \in G | \alpha(a) = a\}\). We call \(\text{stab}(a)\) the stabilizer of \(a\) in \(G\) because it consists of all elements of \(G\) that leave \(a\) fixed. Prove that \(\text{stab}(a)\) is a subgroup of \(G\).

7. Suppose that \(\beta\) is a 10-cycle. For which integers \(i\) between 2 and 10 is \(\beta^i\) also a 10-cycle? (Hint: Apply what you know about cyclic groups and consider the cycle structure of permutations of order 10.)
8. In $S_3$, find elements $\alpha$ and $\beta$ such that $|\alpha| = 2$, $|\beta| = 2$, and $|\alpha \beta| = 3$. How does this relate to our theorem about the order of the product of disjoint cycles?

9. Given that $\beta$ and $\gamma$ are in $S_4$ with $\beta \gamma = (1432)$, $\gamma \beta = (1243)$, and $\beta(1) = 4$, determine $\beta$ and $\gamma$. 
