These are suggested practice problems for the exam, to help you get more familiar with cyclic groups. You do not have to turn them in.

1. In $\mathbb{Z}_{24}$ list all generators for the subgroup of order 8. Let $G = \langle a \rangle$ and let $|a| = 24$. List all generators for the subgroup of order 8.

2. How many subgroups does $\mathbb{Z}_{20}$ have? List a generator for each of these subgroups.

3. List all the elements of order 8 in $\mathbb{Z}_{8000000}$. How do you know your list is complete? Let $a$ be a group element such that $|a| = 8000000$. List all elements of order 8 in $\langle a \rangle$. How do you know your list is complete?

4. Determine the subgroup lattice for $\mathbb{Z}_{12}$. (A subgroup lattice is a chart that includes all subgroups, and uses lines to indicated inclusions. See Chapter 4 in Gallian for an example.)

5. Let $m$ and $n$ be elements of the group $\mathbb{Z}$. Find a generator for the group $\langle m \rangle \cap \langle n \rangle$.

6. Suppose that $G$ is a cyclic group and that 6 divides $|G|$. How many elements of order 6 does $G$ have? If 8 divides $|G|$, how many elements of order 8 does $G$ have? If $a$ is one element of order 8, list the other elements of order 8.