1. Show that every nonzero element of \( \mathbb{Z}_n \) is a unit or a zero-divisor.

2. Describe all zero-divisors and units of \( \mathbb{Z} \oplus \mathbb{Q} \oplus \mathbb{Z} \).

3. A ring element \( a \) is called an idempotent if \( a^2 = a \). Prove that the only idempotents in an integral domain are 0 and 1.

4. Suppose that \( a \) and \( b \) belong to an integral domain. If \( a^5 = b^5 \) and \( a^3 = b^3 \), show that \( a = b \).

5. Show that a finite commutative ring with no zero-divisors and at least two elements has a unity.

6. Give an example of an infinite integral domain that has characteristic 3.

7. Suppose that \( R \) is an integral domain in which \( 20 \cdot 1 = 0 \) and \( 12 \cdot 1 = 0 \). (Recall that \( n \cdot 1 \) means \( 1 + 1 + \cdots + 1 \) with \( n \) terms.) What is the characteristic of \( R \)?

8. Let \( F \) be a field with characteristic 2 and more than two elements. Show that \( (x + y)^3 \neq x^3 + y^3 \) for some \( x \) and \( y \) in \( F \).

9. Let \( F \) be a field of order 32. Show that the only subfields of \( F \) are \( F \) itself and \( \{0, 1\} \).

10. Let \( S = \{a + bi \mid a, b \in \mathbb{Z}, b \text{ is even}\} \). Show that \( S \) is a subring of \( \mathbb{Z}[i] \), but not an ideal of \( \mathbb{Z}[i] \).

11. If an ideal \( I \) of a ring \( R \) contains a unit, show that \( I = R \).

12. Prove that the only ideals of a field \( F \) are \( \{0\} \) and \( F \) itself.

13. Let \( R \) be a ring and let \( I \) be an ideal of \( R \). Prove that the factor ring \( R/I \) is commutative if and only if \( rs - sr \in I \) for all \( r \) and \( s \) in \( R \).